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Switched Capacitor Simulation with Modelica

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Abstract

To simulate switched-capacitor circuits effectively special simulators use the charge-voltage system of equations instead of the current-voltage system. Furthermore, the set of devices is limited. In this paper possibilities are presented to follow this approach in Modelica. An example switched-capacitor library is implemented as well as example circuits.

1 Introduction

Switched-capacitor (SC) networks are often used for the realization of filters, comparators, or integrators. As a simple example of a switched-capacitor circuit c.f. **figure 1**. Depending on the switching frequency resistors with varying resistance can be created. Integrated SC circuits are often much cheaper than conventional IC's [1]. Since in such circuits the switching frequency and the signal frequency often differ considerably, long time simulations are necessary to investigate the circuit behaviour. This is time consuming because small switching intervals with high currents flowing cause small step sizes during the simulation.

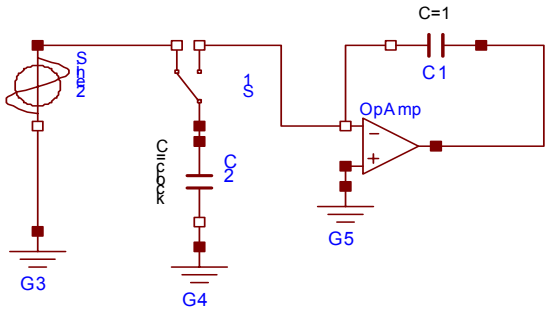

Figure 1: SC Integrator circuit

A simplification is possible if only the voltages at the ends of switching intervals are of interest. In such cases the calculation of the behaviour of currents can be avoided. Provided that the exchange of charges is finished within the switching interval it is sufficient to calculate the voltages for the total charge equalization only. If furthermore only devices are in the circuit which do not need currents explicitely no differentiation of the charge is necessary. In this case only algebraic (linear or nonlinear) equations have to be solved with the stepsize of the switching intervals. Typical devices possible at this switched-capacitor simulation approach are ideal voltage sources, ideal switches, capacitors, voltage-controlled voltage sources, operational amplifiers. Some special simulators exploit this approach, e.g. TOSCA [2], SWITCAP [3], AWEswit [4].

In this paper a possibility is studied to perform the SC-simulation with Modelica.

2 Fundamentals

For the description and simulation of electrical circuits usually relations between currents (i) and voltages (v) are used. The combination of the equations of all devices in a circuit together with the KIRCHHOFF's law equations at nodes results in a current-voltage-system which concsists of differential-algebraic equations (DAE).

This DAE has to be used if the switching behaviour is of interest. If from a more general point of view an abstraction from the switching behaviour is acceptable the switches can be modelled more ideally. In this case high current impulses occur which force the simulation of the DAE to small time steps, and therefore to a poor performance.

A way out is the change from the current-voltage-system to the charge-voltage-system. Provided that:

- the ideal switches are timed in an equidistant scheme (stepsize s)
- the devices are restricted to capacitors and such devices which can be described by algebraic relations between pin voltages only (e.g. ideal voltage sources, voltage controlled voltage sources, ideal operational amplifiers, ideal switches)
- capacitors are the only devices which combine pin currents and pin voltages
- no currents themselves are of interest

Then the current-voltage DAE can be integrated over each step interval of the size s. Since derivations with respect to time occur in capacitors only capacitors are outlined in detail:

The equation of a capacitor is:

$$i = \frac{d}{dt}C(v) \tag{2-1}$$

with i beeing the current and v the voltage over the capacitor. If $[ta, tn]$ with $s = tn - ta$ is the actual time interval the integration of (2-1) with respect to time results into

$$q = \frac{d}{dv}C(v)(v(tn) - v(ta)) \tag{2-2}$$

where q is the charge transported into the capacitor within the interval. If the capacitance is constant the formula is

$$q = C \cdot (v(tn) - v(ta)) \tag{2-3}$$

The device equations (2-2), (2-3) contain voltages and charges only. Since according to our assumtion all other devices can be described by algebraic relations between pin voltages, and KIRCHHOFF's current equations can be trivially integrated using

$$q = \int_{ta}^{tn} i(t)dt \tag{2-4}$$

the resulting system of equations is a linear or nonlinear algebraic **charge-voltage** system.

During simulation it has to be solved once at each time interval at tn. Therefore a higer performance can be expected than solving the current-voltage-DAE. Further optimizations are possible if the network topology is exploited, which often changes between two cases only.

If the restricted amount of devices is left since e.g. a resistor is needed, then the current needs to be calculated by differentiating the charge variable q (2-4). In this case a differential-algebraic system is constructed with the loss of the above mentioned advantages.

There are some generalizations possible such as non-equidistant time grids, switching depending on voltage values and others, which are not yet considered in this paper. In the following the implementaion of a switched-capacitor library is described which bases on the charce-voltage-system of circuit equations.

3 Implementation

In contrast to usual electrical modelling the connectors include the voltage, and the charge, which is transported via the pin in one switching intervall. The charge is a flow value like the current in the current-voltage system since according to (2-4) the charge meets KIRCHHOFF's law. The connector definition is:

```
connector VoltageChargePin
   Modelica.SIunits.Voltage v
               "Potential at the pin";
   flow Modelica.SIunits.Charge q
               "Charge flowing into the pin";
end VoltageChargePin;
```

Using this connector models and partial models can be created like in the Modelica.Electrical.Analog package.

Basically, there are two groups of devices: Devices which depend on the switching interval, like switches and capacitances, and other devices which do not depend on the switching interval. Devices depending on the switching interval must be „informed“ about the events of switching. This could be achieved by further connector, which is connected with a clock generator, or a logic network. Since in this implementation the restriction is that each switch changes at equidistant timesteps, each device depending on switching intervals has a clock parameter with $clock = 2 \cdot s$. Via the sample function

```
algorithm
  when sample(0, clock/2.0) then ...
```

the calculations are controlled which have to be done at switching time points. The advantage of this approach is that no clock connections are necessary. Otherwise the user has to care about the correct clock parameters at each device. This is a disadvantage. In the examples a central clock parameter is introduced. Each device clock parameter is set equal to the central clock parameter by the user. The choice of $clock = 2 \cdot s$ instead of $clock = s$ seems to be practical: the clock parameter covers a complete on-off-interval.

Using this clock handling and equation **(2-3)**, the implementation of the linear constant capacitor device is:

```
model Capacitor
  extends Interfaces.OnePort;
  parameter Modelica.SIunits.Capacitance C=1;
  parameter Real clock=1;
  Real vlast(start=0);
  Real tlast(start=-1);
algorithm
  when sample(0, clock/2) then
    if (time > tlast) then
      tlast := time;
```

```
      vlast := pre(v);
    end if;
  end when;
equation
  q = C*(v - vlast);
end Capacitor;
```

In the algorithm section only once at a sample *tlast* and *vlast* (which correspondend to *ta* and *v(ta)* in equation (2-3)) are calculated. This is ensured by comparing the time with the variable *tlast*. The equation (2-3) itself is located in the equation section because it can be solved manifold at a switching point during iterations.

The switches are modelled similarly. At switching samples the state of the switch is changed. The pin relations are formulated in the equation section:

```
model OpeningSwitch
extends SwitchedCapacitor.Interfaces.OnePort;
   parameter Real clock=1;
   Real s;
   Boolean control(start=false);
   Real tswitch(start=-1);
algorithm
   when sample(0, clock/2.0) then
      if (time > tswitch) then
         tswitch := time;
         control := not control;
      end if;
   end when;
equation
   v = s*(if control then 1 else 0);
   q = s*(if control then 0 else 1);
end OpeningSwitch;
```

If for voltage inputs the electrical models shall be used a converter between the switched-capacitor and the usual electrical domain is necessary. Since signals in the switched-capacitor domain change at switching time points only it is useful to sample the input voltage. The converter model without any feedback into the current-voltage domain is:

```
model ElectricalToSwitchedCapacitorVoltage
   parameter Real clock=1;
   Interfaces.VoltageChargePin pinSC;
   Modelica.Electrical.Analog.Interfaces.Pin
      pinElectrical;
algorithm
   when sample(0, clock/2.0) then
      pinSC.v :=pinElectrical.v;
   end when;
equation
   pinElectrical.i = 0;
end ElectricalToSwitchedCapacitorVoltage;
```

Devices which do not depend on the switching intervals are modelled like the counterparts in the current-voltage-domain. Merely currents (i) are replaced by charges (q). As examples the voltage controled voltage model and the ideal opamp model are cited:

```
model VCV
   extends
      SwitchedCapacitor.Interfaces.TwoPort;
   parameter Real gain=1;
equation
   v2 = v1*gain;
   q1 = 0;
end VCV;

model IdealOpAmp
SwitchedCapacitor.Interfaces.VoltageChargePin
   in_p, in_n, out;
equation
   in_p.v = in_n.v;
   in_p.q = 0;
   in_n.q = 0;
end IdealOpAmp;
```

The device models are combined to a SC-library for test and investigation purposes. It contains simple models only. An extension towards more complicated devices like operational amplifiers with parasitic capacitances and offset or nonlinear capacitance models is possible.

4 Examples

The models of the SC-library are successfully testet at a collection of about 20 circuits:

- simple resistors replaced by switched capacitors
- charging of one or more capacitances
- SC-integrators
- SC delay circuit
- Cauer-filter

In this section some of the examples are presented to demonstrate that the SC-simulation works correctly. All examples were simulated using the simulator Dymola5.3a.

4.1 Constant charge flow

In this example a constant charge flow circuit (**Fig. 2**) is simulated, which is compared with a constant current flow through a resistor.

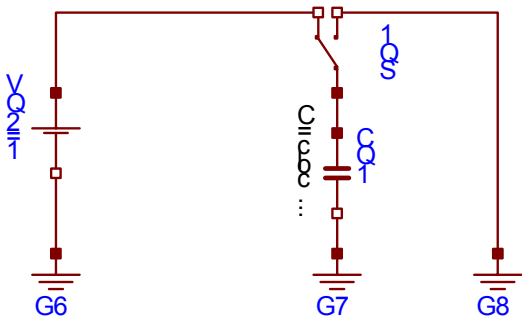

Figure 2: Constant charge flow circuit

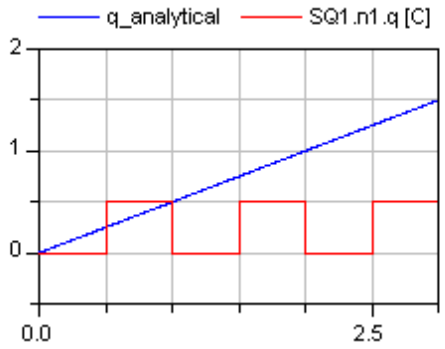

Figure 3: Constant charge flow result

In **Fig. 3** the linearly growing line shows the analytical calculated charge of a circuit like in **Fig. 2** where the switched capacitor with the value *clock/2* is replaced by an equivalent resistor of value *R=2* in the current-voltage domain. After each clock period (=1 second) which includes two switching periods (*s*=0.5 second) the charge flowing into the switch is equal to the charge of 0.5 Coulomb flown in the actual clock interval.

In the following **Fig. 4** the current of the SC-Circuit like **Fig. 2** is simulated in the current-voltage domain. The current peaks depend on the parasitic resistance value in the current-voltage switch model. A purely ideal simulation is not possible in the current-voltage domain.

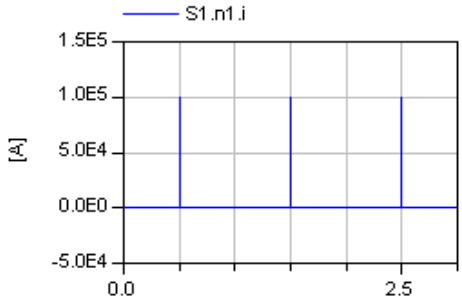

Figure 4: Current peaks in current-voltage simulation

4.2 Charging a capacitance

The following circuit (**Fig. 5**) is a simple charging up of a capacitance. In this example an electrical voltage source is used. The voltage is converted into the SC domain with the charge-voltage system.

The pictures in **Fig. 6** show the voltages of the capacitances *C*, and C_s. Depending on the state of the switch the voltage Cs.p.v of C_s is 1V, if C_s is connected with the voltage source, or it is equal to the voltage C.p.v of *C* which is increasing with each charge equalization.

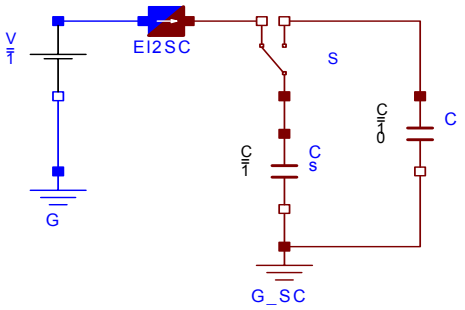

Figure 5: Charging a capacitance

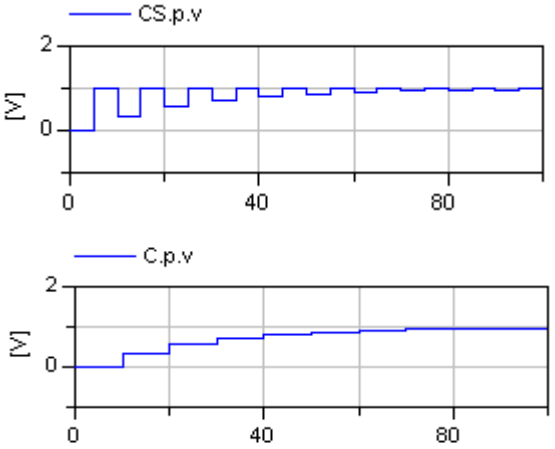

Figure 6: Charging a capacitance: voltages

Furthermore, **Fig. 7** shows the charges flowing through the positive pins of both capacitances.

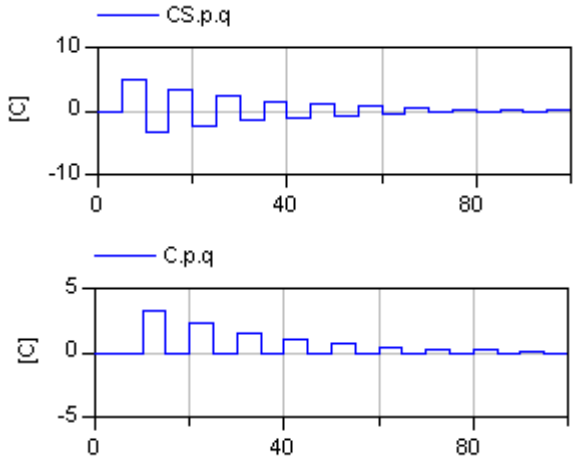

Figure 7: Charging a capacitance: charges

The charge flow through *Cs.p* is positive if C_s is charged by the voltage source. Otherwise it is negative in the case of the charge equalization between both capacitances. The velocity of charging depends on the switching interval length *clock*.

4.3 SC-Integrators

The SC-Integrator [5] according to **Fig. 1** integrates the input voltage. The result which is inverted, can be seen in **Fig. 8**.

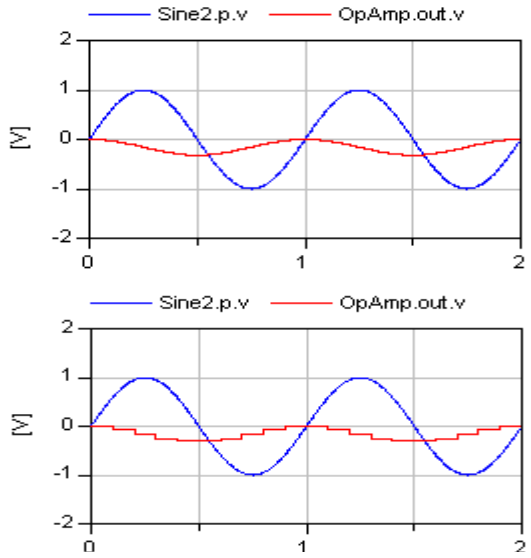

Figure 8: Inverting integration results with clock=0.01 (above) and clock=0.1 (below)

With a slightly changed topology [5] according to **Fig. 9** a noninverting integration of the input signal is possible. The results depend on the clock length. Already large clock switching intervals calculate sufficient results (**Fig. 10**).

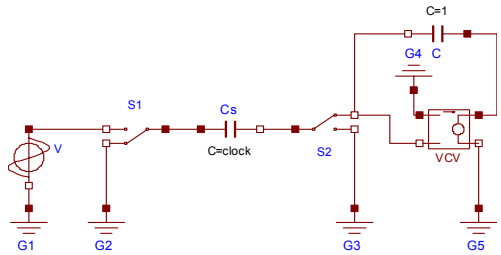

Figure 9: Noninverting SC-Integrator

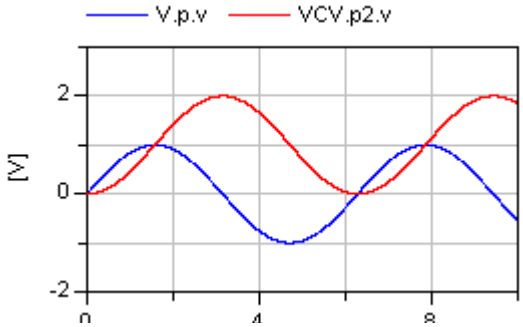

Figure 10: Noninverting integration results

4.4 Delay circuit

A clock-controlled delay example is the circuit in **Fig. 11** which combines two voltage amplifiers [6]. The result is delayed by one clock length. **Fig. 12** shows the input signal and the delayed output at a clock of 1.e-4.

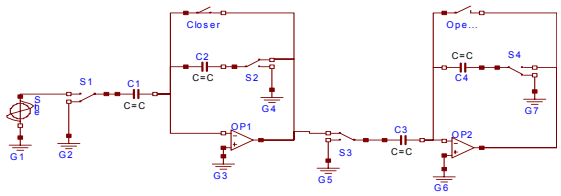

Figure 11: Clock delay circuit

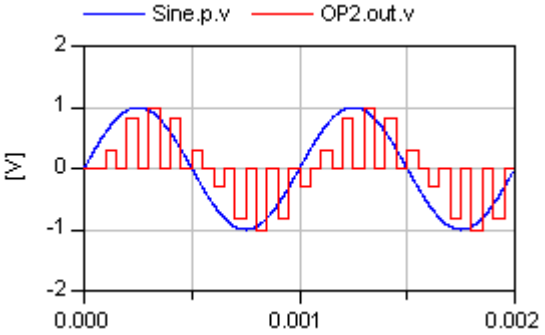

Figure 12: Sine input and delayed SC result

4.5 Cauer filter

As a final and more complex example which demonstrates the possibilities of the SC package, the 5th order cauer filter according to **Fig. 14** is modelled [7]. For purposes of test an unusual time scale is used. The pulse response results of some of the opamp outputs can be seen in **Fig. 13**.

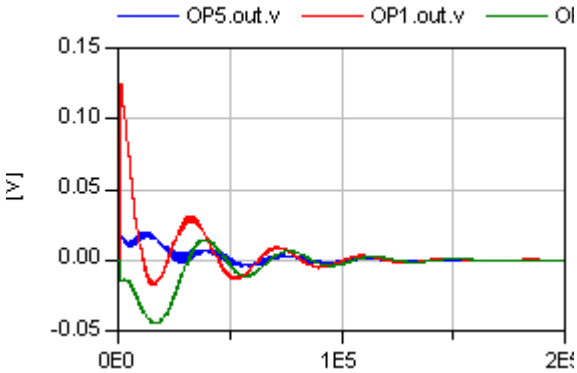

Figure 13: Sine input and delayed SC result

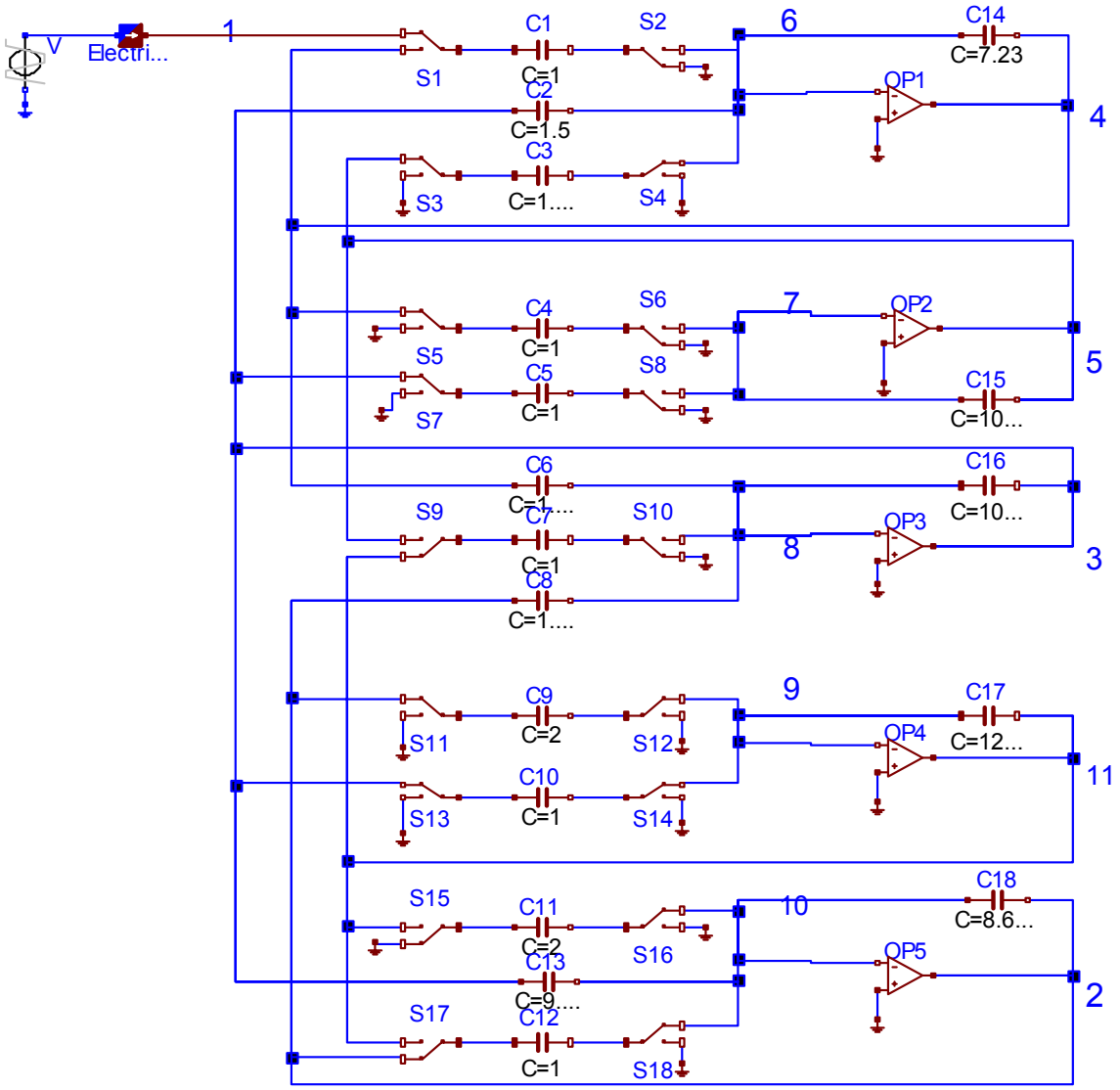

Figure 14: Cauer filter

4.6 Statistics

To compare the performance between simulations of the current-voltage system and the charge-voltage system of SC circuits the following test examples are used:

circuit	
Integ	Integrator according to **Fig. 1**
Integnon	noninverting integrator according to **Fig. 9**
Delay	Delay circuit according to **Fig. 11**
CauerOP	Cauer filter according to **Fig. 14**
CauerVC	like CauerOP, using voltage-controlled voltage sources instead of operational amplifiers

Each circuit is modelled both as current-voltage system using the Modelica.Electrical.Analog package (with default parameters of the switches) and as charge-voltage system using switched capacitor package. In the following the systems are abbreviated with CU (current-voltage system) and CH (charge-voltage system).

In the following table system related quantities calculated by Dymola are collected:

- unkn: unknown variables before translation
- diff: differented variabled befor translation
- tvar: time varying variables after translation
- state: continuous time states after translation

circuit	CU unkn	CU diff	CU tvar	CU state	CH unkn	CH diff	CH tvar	CH state
Integ	40	2	10	2	45	0	14	0
Integnon	61	2	18	2	65	0	24	0
Delay	111	4	35	4	123	0	53	0
CauerOP	382	18	135	14	410	0	209	0
CauerVC	412	18	177	14	440	0	219	0

Table 1: Translation related quantities

In the next table simulation related quantities are shown:

- steps: number of successful steps
- F: number of F-evaluations
- Jac: number of Jacobian-evaluations

circuit	CU steps	CU F	CU Jac	CH steps	CH F	CH Jac
Integ	22744	61244	10800	4700	8900	4200
Integnon	111387	295721	51654	20000	38000	18000
Delay	2613	9205	1173	640	1200	560
CauerOP	19912	241201	15222	5030	9657	4627
CauerVC	23273	331305	20891	5030	9657	4627

Table 2: Simulation related quantities

In the following table the CPU-time is compared:

- tstop: stop time
- CPU: CPU-time for integration in seconds

circuit	tstop	clock	CU CPU	CH CPU
Integ	2	0.01	1.4	0.7
Integinv	10	0.01	5.8	2.0
Delay	0.002	0.0001	1.1	0.95
CauerOP	200000	1000	18.7	1.68
CauerVC	200000	1000	49.2	1.71

Table 3: CPU-time

The simulations run on a 800 MHz PC with 128 MB RAM.

Although in the charge-voltage system the number of time-varying variables after translation is higher than in the current-voltage system (**Table** 1) the computational amount in the charge-voltage system is far less than in the current-voltage system (**Table** 2, **Table** 3). The reason is that the charge-voltage system of equaions is an algebraic one (**Table** 1).

If otherwise the at least necessary number of steps is calculated according to $tstop/(clock/2)$ in **Table** 3 the resulting number is less than the number of steps according to **Table** 2. That means that there are further possibilities of optimization within the simulation algorithm.

5 Conclusion

The main result of this investigation is that switched-capacitor simulation with Modelica and Dymola is possible. The switched-capacitor simulation using the charge-voltage system and the restricted set of devices is clearly faster than the simulation of the current-voltage system. The Cauer example shows that the package can be applied for the simulation of more complex examples than simple test cases. An extension of the package to devices including parasitic effects, nonlinearities etc. is desirable.

Tasks for future research are more flexible controlling of switches e.g. via logic networks and further optimization of the algorithm, especially in comparison with switched-capacitor special simulators.

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