

Multizone Building Model for Thermal Building Simulation in Modelica

Michael Wetter
 United Technologies Research Center
 411 Silver Lane
 East Hartford, CT 06108

Abstract

We present a room model for thermal building simulation that we implemented in Modelica. The room model can be used for controls analysis and energy analysis of one or several rooms that are connected through airflow or heat conduction. The room model can assess energy storage in the air and in the building construction materials, heat transfer between the room and the outside environment and the humidity and CO₂ release to the room air. The humidity storage in the building construction materials is not modeled. We also describe a novel separation of heat transfer mechanisms on which our room model is built on. The separation allowed a significant reduction in model development time, and it allows using state-of-the-art programs for computing prior to the thermal building simulation certain energy flows, such as solar heat gain of an active facade without breaking feedback loops between the HVAC system and the room.

Keywords: Multizone thermal building model, building energy analysis

1 Introduction

Many control design processes require frequency domain analysis, time domain simulations with time steps in the order of seconds or plant model inversion. Commercially available detailed building energy simulation programs, such as EnergyPlus [2], TRNSYS [8], and DOE-2 [20], are not applicable for such analysis. For example, the numerical solution algorithms used in the building envelope model of EnergyPlus, TRNSYS and DOE-2, are all based on a discrete time representation of the building envelope dynamics that does not allow time steps in the order of seconds. In addition, none of the above cited building energy simulation programs can be used as a plant model in a frequency domain controls analysis because they

cannot be integrated into MATLAB/Simulink nor can model that are defined in those programs be automatically inverted. Thus, we developed a physics-based room model in Modelica to allow advanced control design. For the model to be applicable to analyze a large class of problems, we developed it such that it satisfies the following requirements:

1. To be applicable in trade-studies, it is parameterizable with building geometry and material properties.
2. To be applicable for controls design, it captures the building dynamics and the coupling between rooms through air flow and heat conduction in solids.
3. To allow analyzing novel HVAC systems, such as hydronic concrete core cooling systems [9], it allows linking models for HVAC systems including radiant heating and cooling systems to the building.

Developing a detailed thermal building simulation model may take several months if not a year of development time. We therefore identified a model separation approach that allows us to develop a physics-based room model in Modelica at reduced model development time. Our modeling separation approach essentially separates those phenomena that are described by short-wave radiative heat exchange¹ from those phenomena that are described by long-wave radiative heat exchange, heat conduction and heat convection. To describe the state variables of the building envelope and of the room air temperature, the room model uses as input time-series of short-wave heat gains. The short-wave heat gains are *independent* of the state variables, control actions and HVAC

¹By short-wave radiation, we mean radiation whose wave length is in the solar spectrum.

operation, and computing those heat gains requires models that are time-intensive to develop. Since those time-series are independent on the state variables and control actions, they can be precomputed by detailed thermal building simulation programs, such as IDA [14], TRNSYS or EnergyPlus and used as time-dependent disturbances to our room model.

In Section 2, we describe the model separation approach, in Section 3 we describe the equations used to construct the room model and in Section 4 we discuss the implementation in Modelica. A comparative model validation and a comparison between the model implementation in Modelica with a TRNSYS model of comparable complexity can be found in [16].

2 Model Separation Approach

To explain the model separation approach, we will now identify the dominant modes of heat transfer that need to be described adequately to quantify a building's energy consumption and the occupant's thermal comfort [5]. Figure 1 orders the modes of heat transfer on the horizontal axis by the time required to code and test a simulation model that describes the phenomena and on the vertical axis by the uncertainty associated with the modeling assumptions and/or with the model parameters that one typically encounters for an office building. While the exact location of each phenomenon model can be debated, the figure indicates the following characteristics:

- Models with large uncertainty in the modeling assumptions and/or in the model parameters require a short development time.
- Models with a small uncertainty but large development time are those models that describe heat transfer due to short-wave radiation.

Since the short-wave radiation is *independent* on the building's state variables and control actions², the models on the right hand-side of the dashed line in Figure 1 can be computed in a detailed thermal building simulation program, such as EnergyPlus, and their output be used as forcing functions of the Modelica model in the form of time series. Since short-wave radiative heat gains are decoupled from the room air

²An exception could be, for example, if the building occupants close a window blind if the room air temperature gets uncomfortably warm. Our room model could, however, be extended to describe such situations.

and the room's enclosure surface temperatures, there is no loss of accuracy in decoupling the short-wave radiation from the heat transfer by long-wave radiation, conduction and convection.

Furthermore, this model separation does not restrict analysis capabilities such as controls design and system optimization, as the following items illustrate.

- Since all models that depend on the state variables and the control actions are modeled in Modelica, for controls design MATLAB/Simulink need only be interfaced with Modelica but not with a building simulation program.
- Since the HVAC system as well as the building envelope is modeled in Modelica, cost function evaluations during a system optimization does not in general require running a detailed building simulation program. This considerably simplifies the optimization since the numerical noise of state variables with respect to system parameters is hard, if not impossible, to control in most thermal building simulation programs. Numerical noise has been shown to cause optimization algorithms to fail finding an optimal solution in building energy optimization studies [18, 19, 15].

3 Model Description

We will now present the implemented model equations in order to understand what physical phenomena can be analyzed with the room model, and what physical phenomena, such as room air stratification of displacement ventilation, require an extension of the here presented room model in order to be described adequately.

3.1 Distribution of a Room's Solar Gains

To compute the solar gains per unit area for the room surfaces, we assume that all solar radiation that enters the room first hits the floor, and that the floor diffusely reflects the radiation to all other surfaces. We neglect multiple reflections and instead of using view factors between the floor and the other surfaces, we use area-weighted solar distribution factors. Such an approach is common in detailed building simulation programs and has been successfully validated in [15].

Consider a room with $n_f \in \mathbb{N}$ floor area patches, $n_{nf} \in \mathbb{N}$ non-floor area patches, such as walls and windows, and $n_w \in \mathbb{N}$ windows. Let $\mathbf{N}_f \triangleq \{1, \dots, n_f\}$ be the

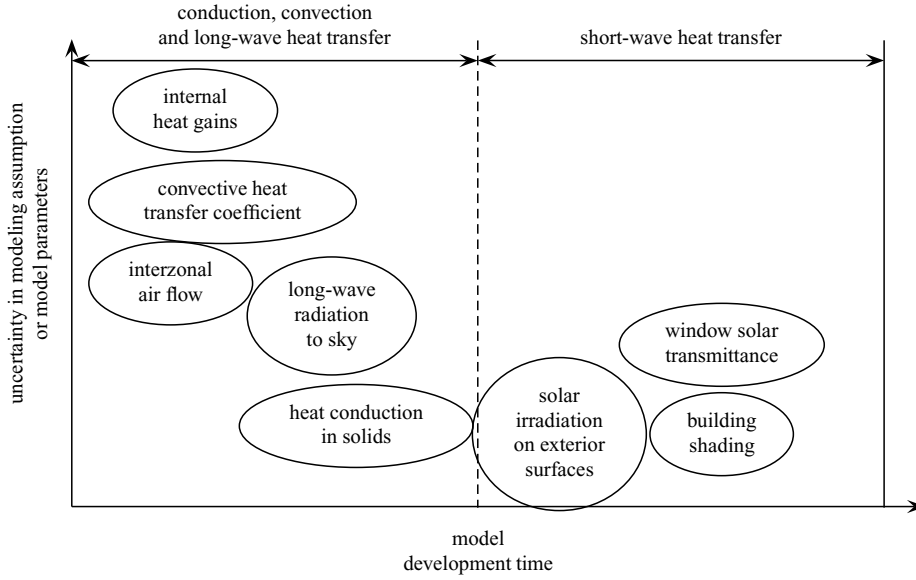


Figure 1: Dominant modes of heat transfer for a thermal building simulation.

set of indices of all floor area patches, let $\mathbf{N}_{nf} \triangleq \{1, \dots, n_{nf}\}$ be the index set of all non-floor surface patches and let $\mathbf{N}_w \triangleq \{1, \dots, n_w\}$ be the set of indices of all windows. Let the solar radiation transmitted from the exterior to the room through all windows be

$$H_{tot}(t) \triangleq \sum_{i \in \mathbf{N}_w} H^i(t) A^i, \quad (1)$$

where $H^i(t)$ denotes the solar radiation that is transmitted per unit area through the i -th window and A^i is the glazed area of the i -th window. The solar radiation that is absorbed by the k -th floor area patch per unit area is

$$q_{SW,abs,0}^k(t) = \frac{\epsilon_{SW}^k}{\sum_{n \in \mathbf{N}_f} A^n} H_{tot}(t), \quad k \in \mathbf{N}_f. \quad (2)$$

The solar radiation that is reflected from the k -th floor area patch is

$$Q_r^k(t) = \frac{A^k (1 - \epsilon_{SW}^k)}{\sum_{n \in \mathbf{N}_f} A^n} H_{tot}(t), \quad k \in \mathbf{N}_f. \quad (3)$$

Therefore, the solar radiation that is reflected by the whole floor area is

$$Q_r(t) = \sum_{n \in \mathbf{N}_f} Q_r^n(t) = \frac{\sum_{n \in \mathbf{N}_f} A^n (1 - \epsilon_{SW}^n)}{\sum_{n \in \mathbf{N}_f} A^n} H_{tot}(t). \quad (4)$$

We distribute $Q_r(t)$ to all non-floor areas, weighted by their solar absorptivity ϵ_{SW} and solar transmissivity for diffuse irradiation τ_{SW} (which is non-zero for windows). We compute the solar radiation that is absorbed

or transmitted by the k -th non-floor surface patch per unit area as

$$q^k(t) = Q_r(t) \frac{\epsilon_{SW}^k + \tau_{SW}^k}{\sum_{m \in \mathbf{N}_{nf}} A^m (\epsilon_{SW}^m + \tau_{SW}^m)}, \quad k \in \mathbf{N}_{nf}. \quad (5)$$

For opaque constructions, $q^k(t)$ is assumed to be completely absorbed and hence $q_{SW,abs,0}^k(t) = q^k(t)$. For windows, however, the absorbed radiation at the room-side pane is

$$q_{SW,abs,0,r}^k(t) = \epsilon_{SW}^k q^k(t), \quad (6)$$

and the remaining fraction of $q^k(t)$ is assumed to be transmitted to the exterior.³

3.2 Distribution of Internal Heat Gains

The radiative and sensible convective and latent heat gains caused by appliances, such as computers, and by people are defined by time series and are input to this model. For values between the support points of the time series, we use an interpolation to define the heat gains as a Lipschitz continuously differentiable function of time.⁴

The radiative internal heat gains are distributed to the room enclosing surfaces as follows: For some $n \in \mathbb{N}$,

³A fraction of $(1 - \epsilon_{SW}^k) q^k(t)$ would be absorbed by the exterior pane, but we neglect this effect.

⁴Using smooth functions to describe the heat gains can significantly reduce the computation time since smoothing eliminates time events.

let $\mathbf{N} \triangleq \{1, \dots, n\}$ denote the index set of all room surfaces. For radiative internal heat gains, the i -th surface is assumed to absorb the heat gain per unit area

$$q_{LW,abs,0}^i(t) = \frac{\varepsilon_{LW}^i}{\sum_{k \in \mathbf{N}} \varepsilon_{LW}^k A^k} Q_{int,LW}(t), \quad (7)$$

where ε_{LW} denotes the absorptivity for long-wave radiation, and $Q_{int,LW}(t)$ denotes the radiative internal heat gain of the zone. Since the long-wave emissivity of window glass is close to unity, we consider windows to be opaque in the long-wave radiation spectrum.

3.3 Humidity Gains

The humidity gain $\dot{m}_{H_2O}(t)$ in the room is computed as

$$\dot{m}_{H_2O}(t) = \frac{Q_{lat}(t)}{r_0 + c_p T}, \quad (8)$$

where $Q_{lat}(t)$ is the room's latent heat gain, $r_0 = 2501.6 \cdot 10^3 \text{ J/kg}$ is the enthalpy of evaporation of water vapor at 0°C , $c_p = 1860 \text{ J/(kg K)}$ is the specific heat capacity of water vapor and T is the temperature at which the water vapor is released. We assume that the temperature at which the water vapor is released is the average body surface temperature of a human, which is 34°C [1, p 8.1].

3.4 CO₂ Gains

Based on the room's latent heat gains, we compute the number of room occupants in the room and then, using the CO₂ release per person, we compute the CO₂ release in the room. Table 3.4 is a partial list of latent heat gains of occupants of conditioned spaces, adopted from the ASHRAE Fundamentals [1, p 28.8]. The number of people $n_p(t)$ is computed as

$$n_p(t) = \frac{Q_{lat}(t)}{q_{lat}}, \quad (9)$$

where $Q_{lat}(t)$ is the room's latent heat gain and q_{lat} is the latent heat gain per person, which is a user specified parameter and can be obtained from Table 3.4.

Table 3.4 is adopted from [10] and specifies the CO₂ emissions of a person at different degrees of activity. Using the emitted mass flow of CO₂ per person and the number of occupants in the room as computed in (9), we compute the total CO₂ source in the room as

$$\dot{m}_{CO_2}(t) = n_p(t) \dot{m}'_{CO_2}. \quad (10)$$

3.5 Radiative Heat Exchange between Room Surfaces

Let $\mathbf{N} \triangleq \{1, \dots, n\}$ be as above. To compute the radiative heat transfer between the room surfaces, we define a radiation temperature $T^*(t)$ as

$$T^*(t) \triangleq \frac{\sum_{k \in \mathbf{N}} \varepsilon_{LW}^k A^k T_{sur}^k(t)}{\sum_{k \in \mathbf{N}} \varepsilon_{LW}^k A^k}, \quad (11)$$

where $T_{sur}^k(t)$ is the surface temperature of the k -th surface patch. We assume that each surface only exchanges radiation with an imaginary surface of much larger area which is at temperature $T^*(t)$. The radiative heat transfer to the surface $k \in \mathbf{N}$ can be described by

$$q_{LW}^k(t) = \sigma \varepsilon_{LW}^k (T^*(t)^4 - T^k(t)^4), \quad (12)$$

where $\sigma = 5.670 \cdot 10^{-8} \text{ J/(K}^4 \text{ m}^2 \text{ s)}$ is the Stefan-Boltzmann constant. To reduce computation time, we linearize the nonlinear equation (12) around $T_0 = 273.15 \text{ K}$. By introducing a linearized heat transfer coefficient, defined as

$$h_{LW}^k \triangleq 4\sigma \varepsilon_{LW}^k T_0^3, \quad (13)$$

we can write

$$q_{LW}^k(t) = h_{LW}^k (T^*(t) - T^k(t)). \quad (14)$$

Equations (14) and (11) are consistent with conservation of energy because

$$\begin{aligned} \sum_{k \in \mathbf{N}} A^k q_{LW}^k(t) &= 4\sigma T_0^3 \sum_{k \in \mathbf{N}} \varepsilon_{LW}^k A^k (T^*(t) - T^k(t)) \\ &= 4\sigma T_0^3 \left(\sum_{k \in \mathbf{N}} \varepsilon_{LW}^k A^k T^*(t) - \sum_{k \in \mathbf{N}} \varepsilon_{LW}^k A^k T^k(t) \right) \\ &= 0. \end{aligned} \quad (15)$$

3.6 Room Air

We assume the room air to be completely mixed. This is a common model assumption in thermal building simulation programs and is used, for example, in [2, 8, 14, 20]. If the model were to be used for displacement ventilation, it would have to be modified to take into account the air stratification [13]. The time rate of change of the room air temperature is

$$\begin{aligned} m_r c_p \frac{dT_{air}(t)}{dt} &= Q_{int,con}(t) + Q_{aux,con}(t) \\ &+ \sum_{i=1}^m \dot{m}^i(t) c_p (T^i(t) - T_{air}(t)) \\ &+ \sum_{i=1}^n A^i h_{con,0}^i (T_{sur}^i(t) - T_{air}(t)), \end{aligned} \quad (16)$$

Table 1: Latent heat gains from occupants of conditioned spaces. Partial list adopted from ASHRAE Fundamentals [1, p 28.8].

Degree of activity	Typical usage	Latent heat q_{lat} in [W/person]
Seated, very light work	Offices, hotels, apartments	45
Moderately active office work	Offices, hotels, apartments	55
Standing, light work; walking	Department store; retail store	55
Walking, standing	Drug store; bank	70

 Table 2: CO₂ emissions per person at different degrees of activity. Partial list adopted from [10].

Degree of activity	CO ₂ emissions \dot{V}'_{CO_2} in [dm ³ /h/person]	CO ₂ emissions \dot{m}'_{CO_2} in [kg/s/person]
Resting	12	0.00582
Sedentary, such as reading or writing	15	0.00728
Light work, standing, such as laboratory work, typesetting	23	0.0112

where m_r is the mass of the room volume, c_p is the air's specific heat capacity and $Q_{int,con}(t)$ is the internal convective heat gain. The term $Q_{aux,con}(t)$ represents convective heat gains caused by an auxiliary heating/cooling system that can be modeled outside of the room model, such as an electric baseboard heater. The term $\sum_{i=1}^m \dot{m}^i(t) c_p (T^i(t) - T_{air}(t))$ describes air flow into the room caused, for example, by the HVAC system and by interzonal air exchange. Any number of HVAC systems or interzonal air exchange models can be connected to the room. This makes the room model applicable for analyzing coupling between rooms due to static pressure difference or buoyancy driven air flow. The term $\sum_{i=1}^n A^i h_{con,0}^i (T_{sur}^i(t) - T_{air}(t))$ describes the convective heat transfer between the room air and the room's surface temperature. In (16), we assumed the convective heat transfer coefficient to be constant, but the model could be replaced by a model which computes the convective heat transfer coefficient as a function of the temperature difference or as a function of the air velocity near the surface.

3.7 Heat Conduction in Solids

Modeling the dynamics of the building envelope is important since the building envelope is typically a building's biggest thermal storage element which can be exploited for reducing cooling peak demand and energy.

The heat conduction in an opaque construction of thickness $L > 0$ over a time interval $(0, \tau)$ is computed using the Fourier equation

$$\frac{\partial}{\partial z} \left(k(z) \frac{\partial T_{sol}(z,t)}{\partial t} \right) = C(z) \frac{\partial T_{sol}(z,t)}{\partial t}, \quad (17)$$

on $(z,t) \in (0, L) \times (0, \tau)$, where $z \in (0, L)$ is the spatial coordinate, $k(z)$ is the thermal conductivity and $C(z) \triangleq \rho(z) c(z)$ is the thermal heat capacity per unit volume. For composite constructions, the material properties $k(\cdot)$ and $C(\cdot)$ typically vary from one layer to another and hence are step functions of the spatial coordinate z .

The boundary conditions are at the room-side surface (at $z = 0$)

$$\begin{aligned} -k(0) \frac{\partial T_{sol}(0,t)}{\partial z} &= h_{con,0} (T_{air}^0(t) - T_{sol}(0,t)) \\ &\quad + q_{SW,abs,0}(t) + q_{LW,abs,0}(t) \end{aligned} \quad (18)$$

and at the exterior surface (at $z = L$)

$$\begin{aligned} k(L) \frac{\partial T_{sol}(L,t)}{\partial z} &= h_{con,1} (T_{air}^1(t) - T_{sol}(L,t)) \\ &\quad + q_{SW,abs,1}(t) + q_{LW,abs,1}(t), \end{aligned} \quad (19)$$

where $h_{con,0}$ and $h_{con,1}$ are the convective heat transfer coefficients, $T_{air}^0(t)$ and $T_{air}^1(t)$ are the air temperatures

near the two surfaces, $q_{SW,abs,0}(t)$ and $q_{SW,abs,1}(t)$ are the short-wave radiation absorbed by the room-side and exterior surface, and $q_{LW,abs,0}(t)$ and $q_{LW,abs,1}(t)$ are the long-wave radiation absorbed by the room-side and exterior surface.

To compute a numerical approximation to the solution of (17), we spatially discretize (17) using a finite difference scheme.⁵ In composite constructions, the material properties of adjacent layers, say in material 1 and material 2, can be so that $k_1/k_2 \approx 10^{-1}$, $C_1/C_2 \approx 10^2$, and hence the ratio of the thermal diffusivities $\alpha \triangleq k/C$ can be $\alpha_1/\alpha_2 \approx 10^1$. Thus, the thermal diffusivity in adjacent layers can vary by an order of magnitude. Hence, if the spatial grid generation does not account for the material properties, then the time rate of change of the different nodes can be significantly different from each other, which can cause the system of ordinary differential equations to be stiff. Thus, we give the user the option to generate the spatial grid automatically so that under the assumption of equal heat transfer, each node temperature has a similar time rate of change. The automatic grid generation is done as follows.

From dimensionless analysis, one can obtain a characteristic time, called the *Fourier* number, as

$$Fo \triangleq \frac{\alpha t}{L^2}, \quad (20)$$

where α denotes the thermal diffusivity, t denotes time and L denotes the characteristic length [7]. It is favorable to generate the spatial grid so that the ratio (t/Fo) is equal to an arbitrary constant Π , which we define as

$$\Pi \triangleq \left(\frac{t}{Fo}\right)^{1/2} = \frac{L}{\sqrt{\alpha}}. \quad (21)$$

Now, let $K \in \mathbb{N}$ denote the number of material layers in an opaque composite construction, and let $\{l^k\}_{k=1}^K$ denote the thickness of each material layer. In view of (21), we compute the time constant of each material layer as

$$\Pi^k = \frac{l^k}{\sqrt{\alpha^k}}, \quad k \in \{1, \dots, K\}, \quad (22)$$

and we compute the estimated number of compart-

⁵Since the coefficients of (17) are step functions, the partial differential equation has a weak but no classical solution for composite constructions. However, using a finite difference scheme is customary in thermal building simulation as it requires less development time than a finite element model.

ments $\hat{N} \in \mathbb{R}$ for each layer as⁶

$$\hat{N}^k = N_{ref} \frac{\Pi^k}{\Pi_{ref}}, \quad (23)$$

where $N_{ref} \in \mathbb{N}$ is a user-specified number of compartments for a reference layer, which we define as a concrete layer with thickness $L_{ref} \triangleq 0.20$ m and thermal diffusivity $\alpha_{ref} \triangleq 3.64 \cdot 10^{-7}$ m²/s. Hence, $\Pi_{ref} \triangleq L_{ref}/\sqrt{\alpha_{ref}} = 331.4$ s^{1/2}. We compute the number of compartments for each material layer as $N^k = \lceil \hat{N}^k \rceil$, with $k \in \{1, \dots, K\}$, where the notation $\lceil \cdot \rceil$ is defined for $s \in \mathbb{R}$ as $\lceil s \rceil \triangleq \min\{k \in \mathbb{Z} \mid k \geq s\}$. Then, we divide each material layer $k \in \{1, \dots, K\}$ in compartments of length $\Delta^k \triangleq l^k/N^k$. Thus, the total number of compartments in the construction is $N = \sum_{k=1}^K N^k$.

3.8 Window Model

Detailed window models, such as the ones in EnergyPlus, consist of a model that computes the optical properties and a model that solves the heat balance equations. In the optical model, the solar transmittance, absorbance and reflectance of each window pane is computed first for direct irradiation at various incidence angles and then for hemispherical (diffuse) irradiation as if each pane would stand alone. Next, those data are used to compute the transmittance, absorbance and reflectance of each pane, but now taking into account multiple reflections between the individual panes and between the window and possible shading devices. Finally, those properties are passed from the optical to the heat balance model as a function of the solar incidence angle so that the heat balance model can compute the panes' temperature, long-wave radiative, conductive and convective heat transfer. Developing such models is time-intensive, see for example [15] for a detailed model description. We therefore implemented a simplified model for double pane windows that uses the solar radiation $H(t)$ that is transmitted per unit area through the window from the outside to the room as an input to a heat balance model. This transmitted solar radiation can be obtained from building energy simulation programs such as EnergyPlus in the form of a time series. The biggest heat gain of the room due to solar radiation is typically caused by the transmitted solar radiation, rather than by the solar radiation that is absorbed by the window and then conducted through the glass and convected to the room air. Therefore, we expect that the simplified model described below

⁶We use \hat{N} as a real number and will round later to an integer.

is accurate enough for many room configurations, except for rooms with a high ratio of glass to wall area.

We will use the same notation as in the Window 4 software manual [6]. In particular, “N,1” denotes that the radiation strikes the window from the outside pane “N” and is transmitted through the inside pane “1”, and “N,N” denotes that the radiation strikes the outside pane and is absorbed by the outside pane. The superscript “ \perp ” denotes irradiation normal to the window pane and the subscript “inc” denotes incident radiation. We use the following data as input to our optical model:

1. A time series, describing the transmitted solar radiation $H(t)$ per unit area.
2. The solar transmittance from the outside to the room for normal irradiation, defined as

$$T_{SW}^{N,1,\perp} \triangleq \frac{H^\perp}{H_{inc}^\perp}. \quad (24)$$

3. The solar absorptance for the room-side window pane for normal irradiation from the outside, defined as

$$A_{SW}^{N,1,\perp} \triangleq \frac{q_{SW,abs,0}^\perp}{H_{inc}^\perp}. \quad (25)$$

4. The solar absorptance for the exterior window pane for normal irradiation from the outside, defined as

$$A_{SW}^{N,N,\perp} \triangleq \frac{q_{SW,abs,1}^\perp}{H_{inc}^\perp}. \quad (26)$$

The transmittance and absorptance are functions of the incidence angle ϕ . We assume that the functional dependence of transmittance and absorptance is similar, i.e., we assume that there exists a function $g: [0, 90^\circ] \rightarrow \mathbb{R}$, such that for any $\phi \in [0, 90^\circ]$,

$$T_{SW}^{N,1}(\phi) = T_{SW}^{N,1,\perp} g(\phi), \quad (27)$$

$$A_{SW}^{N,1}(\phi) = A_{SW}^{N,1,\perp} g(\phi), \quad (28)$$

$$A_{SW}^{N,N}(\phi) = A_{SW}^{N,N,\perp} g(\phi). \quad (29)$$

Since the radiation that is absorbed in an infinitesimal glass element of thickness ds is proportional to the radiation that is transmitted through ds [12], assuming the existence of such a function $g(\cdot)$ is reasonable. Under this assumption, we can compute the absorbed radiation at the room-side window pane as

$$q_{SW,abs,0}(t) = \frac{H(t)}{T_{SW}^{N,1,\perp}} A_{SW}^{N,1,\perp} \quad (30)$$

and at the exterior window pane as

$$q_{SW,abs,1}(t) = \frac{H(t)}{T_{SW}^{N,1,\perp}} A_{SW}^{N,N,\perp}. \quad (31)$$

Note that (30) accounts only the absorption of the short-wave radiation from the outside to the room-side. The absorption of the short-wave radiation that is reflected by the room and radiated to the outside is described in (6).

In the thermal model, we neglect the window’s thermal capacity since it is much smaller than the thermal capacity of the building envelope. This assumption is customary in thermal building simulation programs.

The heat balance of the two glass layers is

$$\begin{aligned} 0 = & h_{con,0} (T_{air}(t) - T_{win}^1(t)) \\ & + q_{SW,abs,0}(t) + q_{SW,abs,0,r}(t) \\ & + q_{LW,abs,0}(t) + U (T_{win}^2(t) - T_{win}^1(t)) \end{aligned} \quad (32)$$

and

$$\begin{aligned} 0 = & h_{con,1} (T_{out}(t) - T_{win}^2(t)) + q_{SW,abs,1}(t) \\ & + q_{LW,abs,1}(t) + U (T_{win}^1(t) - T_{win}^2(t)), \end{aligned} \quad (33)$$

where $T_{win}^1(t)$ and $T_{win}^2(t)$ are the room side and exterior side glass temperatures, $q_{LW,abs,0}(t)$ and $q_{LW,abs,1}(t)$ are the long-wave radiation that is absorbed by the room side and the exterior side, $q_{SW,abs,0,r}(t)$ is as in (6) and U is the window gap’s heat transfer coefficient. We assume U to be independent of the temperature difference $T_{win}^2(t) - T_{win}^1(t)$ since conduction through stagnant air is the dominant heat transport mechanism for air or argon gaps up to 10 mm thickness [3].

3.9 Long-wave Radiation between Exterior Surfaces and Environment

Exterior surfaces of opaque constructions and of windows participate in long-wave radiative heat exchange with the sky and the environment. The sky temperature $T_{sky}(t)$ is an input to the Modelica room model. We assume that the sky temperature is defined as the temperature that results in the same long-wave radiative heat exchange as if the sky would be a black body at temperature $T_{sky}(t)$ ⁷. We assume that the ground is at the temperature that is connected to the exterior heat flux port of the construction, which is typically the outside air temperature. The view factor φ_{s-g} between the

⁷The black body sky temperature can be obtained, for example, from EnergyPlus.

surface and the ground is assigned as

$$\varphi_{s-g} = \begin{cases} 1 & , \text{ for floors,} \\ 1/2 & , \text{ for walls,} \\ 0 & , \text{ for ceilings.} \end{cases} \quad (34)$$

Let $T_{ext}(t)$ denote the ground temperature. We define an environment temperature as

$$T_{env}(t) \triangleq (\varphi_{s-g} T_{ext}(t)^4 + (1 - \varphi_{s-g}) T_{sky}(t)^4)^{1/4}, \quad (35)$$

and we define a radiative heat transfer coefficient as

$$h_{LW}(t) \triangleq 4\sigma\epsilon_{LW} \left(\frac{T_{sur}(t) + T_{env}(t)}{2} \right)^3, \quad (36)$$

where $\sigma = 5.670 \cdot 10^{-8} \text{ W/m}^2/\text{K}^4$ is the Stefan-Boltzmann constant and $T_{sur}(t)$ is the exterior surface temperature of the construction. The long-wave radiative heat flux is computed as

$$q_{LW}(t) = h_{LW}(t) (T_{env}(t) - T_{sur}(t)). \quad (37)$$

4 Implementation in Modelica

We implemented the building model using Modelica 2.2 [11] and Modelica_Fluid [4]. Figure 2 shows the graphical view of the room model implementation in Dymola. There is a FluidPort (labeled “fluPor”) and a HeatPort (labeled “heaPor”) that connect to a JunctionVolume that models the room air (labeled “air”). The fluid port can be used to attach an airflow network that defines a model for an air-conditioning system or for interzonal air exchange [17]. The heat port can be used to add a convective heat source or sink to the room. There are models for interior surfaces “intSur” whose temperature and heat flow can be connected to an external model that may, for example, model heat transfer inside a radiant heating slab. The room-facing surface then automatically participates in the radiative heat transfer balance described in Section 3.5. The models “airCon” allows constructions to be modeled whose exterior surface have a prescribed air temperature as boundary condition, such as a floor above a crawl space. The models “parCon” describe the behavior of partition constructions, i.e., constructions where both surfaces are exposed to the same room. The models “extCon” and “winCon” describe exterior opaque constructions and windows. The models “intGai”, “qLatCon” and “intConGai” convert internal gains to heat, water vapor and carbon-dioxide flows. The models whose icons have a yellow bar are

replaceable placeholders for models that may be replaced at compile time with an array of models that have the actual physical implementation.

5 Conclusions

Our model separation approach led to shorter model development time and allows our model to be used with state-of-the-art software for daylighting simulation. The developed room model captures the dynamics required for controls design, and the energy flows required for building energy analysis simulations.

6 Acknowledgments

This research was supported by the U.S. Department of Commerce, National Institute of Standards and Technology, Advanced Technology Program under the agreement number 70NANB4H3024.

7 Nomenclature

7.1 Conventions

1. Vectors elements are denoted by superscripts.
2. $f(\cdot)$ denotes a function where (\cdot) stands for the undesignated variables. $f(x)$ denotes the value of $f(\cdot)$ for the argument x .

7.2 Symbols

$a \in A$	a is an element of A
\mathbb{N}	$\{1, 2, 3, \dots\}$
\triangleq	equal by definition

References

- [1] ASHRAE. *Fundamentals*. American Society of Heating, Refrigeration and Air-Conditioning Engineers, 1997.
- [2] Drury B. Crawley, Linda K. Lawrie, Frederick C. Winkelmann, Walter F. Buhl, Y. Joe Huang, Curtis O. Pedersen, Richard K. Strand, Richard J. Liesen, Daniel E. Fisher, Michael J. Witte, and Jason Glazer. EnergyPlus: creating a new-generation building energy simulation program. *Energy and Buildings*, 33(4):443–457, 2001.

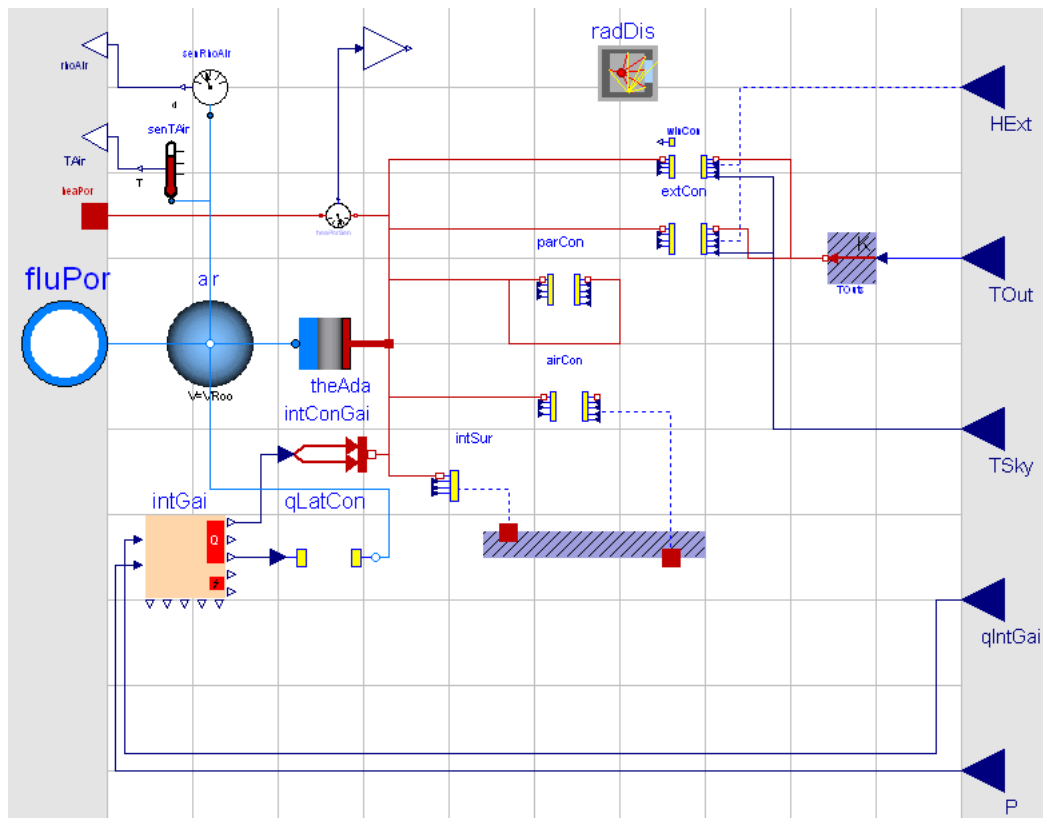


Figure 2: View of room model in Dymola's graphical editor.

- [3] Bernard R. Cuzzillo and Patrick J. Pagni. Thermal breakage of double-pane glazing by fire. *J. of Fire Prot. Engr.*, 9(1):1–11, 1998.
- [4] Hilding Elmqvist, Hubertus Tummescheit, and Martin Otter. Object-oriented modeling of thermo-fluid systems. In Peter Fritzson, editor, *Proceedings of the 3rd Modelica conference*, pages 269–286, Linköping, Sweden, November 2003. Modelica Association and Institutionen för datavetenskap, Linköpings universitet.
- [5] Ole P. Fanger. *Thermal Comfort*. McGraw Hill, 1973.
- [6] E. U. Finlayson, D. K. Arasteh, C. Huizenga, M. D. Rubin, and M. S. Reilly. WINDOW 4.0: Documentation of calculation procedures. Technical Report LBL-33943, Lawrence Berkeley National Laboratory, Berkeley, CA, USA, July 1993.
- [7] Jack Philip Holman. *Heat Transfer*. McGraw-Hill, 8 edition, 1997.
- [8] S. A. Klein, J. A. Duffie, and W. A. Beckman. TRNSYS – A transient simulation program. *ASHRAE Transactions*, 82(1):623–633, 1976.
- [9] Markus Koschenz and Viktor Dorer. Interaction of an air system with concrete core conditioning. *Energy and Buildings*, 30(2):139–145, June 1999.
- [10] R. Lochau. *Handbuch der Klimatechnik*, volume I, chapter Physiologische Grundlagen, pages 61–104. C.F. Müller GmbH, Karlsruhe, 1989.
- [11] Modelica Association. *Modelica – A Unified Object-Oriented Language for Physical Systems Modeling, Language Specification, Version 2.2*, February 2005.
- [12] Michael F. Modest. *Radiative Heat Transfer*. McGraw-Hill, Inc., 1993.
- [13] Simon J. Rees and Philip Haves. A model of a displacement ventilation system suitable for system simulation. In *Proc. of the 4-th IBPSA Conference*, pages 199–205, Madison, Wisconsin, USA, August 1995.

- [14] Per Sahlin and Axel Bring. IDA solver – A tool for building and energy systems simulation. In J. A. Clarke, J. W. Mitchell, and R. C. Van de Perre, editors, *Proc. of the IBPSA Conference*, Nice, France, August 1991.
- [15] Michael Wetter. *Simulation-Based Building Energy Optimization*. PhD thesis, University of California at Berkeley, 2004.
- [16] Michael Wetter. Modelica versus TRNSYS – a comparison between an equation-based and a procedural modeling language for building energy simulation. Submitted to: *SimBuild 2006 Conference*, <http://ceae.colorado.edu/ibpsa/SimBuild06/index.html>, August 2006.
- [17] Michael Wetter. Multizone airflow model in modelica. Submitted to: *Modelica 2006 conference*, <http://www.modelica.org/events/modelica2006/>, September 2006.
- [18] Michael Wetter and Jonathan Wright. Comparison of a generalized pattern search and a genetic algorithm optimization method. In G. Augenbroe and J. Hensen, editors, *Proc. of the 8-th IBPSA Conference*, volume III, pages 1401–1408, Eindhoven, NL, August 2003.
- [19] Michael Wetter and Jonathan Wright. A comparison of deterministic and probabilistic optimization algorithms for nonsmooth simulation-based optimization. *Building and Environment*, 39(8):989–999, August 2004.
- [20] F. C. Winkelmann, B. E. Birsdall, W. F. Buhl, K. L. Ellington, A. E. Erdem, J. J. Hirsch, and S. Gates. DOE-2 supplement, version 2.1E. Technical Report LBL-34947, Lawrence Berkeley National Laboratory, Berkeley, CA, USA, November 1993.